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Optimal Procurement Mechanisms for Divisible Goods with Capacitated Suppliers

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Abstract

The literature on procurement auctions typically assumes that the suppliers are uncapacitated (see, e.g. Dasgupta and Spulber, 1990; Che, 1993). Consequently, these auction mechanisms award the contract to a single supplier. We study mechanism design in a model where suppliers have limited production capacity, and both the marginal costs and the production capacities are private information. We provide a closed form solution for the revenue maximizing direct mechanism when the distribution of the cost and production capacities satisfies a modified *regularity* condition (Myerson, 1981). We also present a sealed low bid implementation of the optimal direct mechanism for the special case of identical suppliers, i.e. symmetric environment.

The results in this paper extend to other principle-agent mechanism design problems where the agents have a privately known upper bound on allocation. Examples of such problems include monopoly pricing with adverse selection and forward auctions.

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1 Background and Motivation

Using auctions to award contracts to supply goods and services is now pervasive in many industries, e.g. electronics industry procurements, government defence procurements (Naegelen, 2002; Dasgupta and Spulber, 1990, and references therein), supply chain procurements (Chen, 2004, and references therein). Since the auctioneer is the *buyer*, the bidders are the *suppliers* or *sellers*, and the object being auctioned is the right to supply, these procurement auctions are also called *reverse auctions*. The use of reverse auctions to award contracts has been vigorously advocated since competitive bidding results in lower procurement costs, facilitates demand revelation, allows order quantities to be determined ex-post based on the bids and limits the influences of nepotism and political ties. Moreover, the advent of the Internet has significantly reduced the transaction costs involved in conducting such auctions. There is now a large body of literature detailing the growing importance of reverse auctions in industrial procurement. According to Parente et al. (2001), the total value of the B2B online auction transactions totaled 109 billion in 1999, and that number was expected to grow to 2.7 trillion by 2004.

Although auction design is a well-studied problem, the models analyzed thus far do not adequately address the fact that the private information of the bidders is typically multi-dimensional (cost, capacity, quality, lead times, etc.) and the instruments available to the auctioneer to screen this private information is also multidimensional, e.g. multiple products, multiple components, different procurement locations, etc. This paper investigates mechanism design for a one-shot reverse auction with divisible goods and *capacitated* suppliers, i.e. suppliers with finite capacities. The production capacities, in addition to the production costs, are only known to the respective suppliers and need to be screened by an appropriate mechanism. Thus, in our model the private information of the supplier is two dimensional. However, we assume that the suppliers can only *underbid* capacity. We show how to construct the optimal revenue maximizing direct mechanism for this model. Although the general Bayesian mechanism design problem with 2-dimensional types which is known to be hard, we are able to circumvent the difficulties in the general problem by exploiting the specific structure of the model, in particular that the suppliers are only allowed to underbid capacity. The basic insight is that the optimal mechanism does not give any information rent to a supplier for revealing capacity information when the production cost is known. We also present a low bid implementation of the optimal auction in a symmetric environment.

The paper is organized as follows. In §1.1 we discuss some of the relevant literature. In § 2 we describe the model preliminaries. In this section, we also elaborate on the suppliers' incentive to lie about capacity and consider various special cases of the procurement auction problem. In § 3 we present the optimal direct auction mechanism and its implementation via "pay as you bid" reverse auction. In § 4 we discuss limitations of our model and directions for future research.

1.1 Literature Review

Myerson (1981) first used the indirect utility approach to derive the optimal auction in an *independent private value* (IPV) model. Che (1993) considers 2-dimensional (reverse) auction where the sellers bid price and quality, and the buyer's utility is a function of both quality and price. However, in this model only the costs are private information; thus, the bidder type space is one-dimensional. Also, Che (1993) only considers sourcing from a single supplier; therefore, the problem reduces to one of determining the winning probability instead of the expected allocation. Naegelen (2002) models reverse auctions for department of defense (DoD) projects by a model where the quality of each of the firms is fixed and is common knowledge. The preference over quality in this setting results in virtual utilities which are biased. Again, she only considers the single winner case.

Dasgupta and Spulber (1990) consider a model very similar to the one discussed in this paper except that the suppliers have unlimited capacity. They construct the optimal auction mechanism for both single sourcing and multiple sourcing (due to non-linearities in production costs) when the private information is one-dimensional. Chen (2004) presents an alternate two-stage implementation for the optimal mechanism in Dasgupta and Spulber (1990). In this alternate implementation the winning firm is first determined via competition on fixed fees, and then the winner is offered an optimal price-quantity schedule.

Laffont et al. (1987) solve the optimal nonlinear pricing (single agent principle-agent mechanism design) problem with a two-dimensional type space. They explicitly force the integrability conditions on the gradient of the indirect utility function. Surprisingly, the optimal pricing mechanism (the bundle menus) is rather involved even when the prior is uniform. Rochet and Stole (2003) also provide an excellent survey of multi-dimensional screening and the associated difficulties.

Vohra and Malakhov (2004) describe the indirect utility approach in multi-dimensional discrete type spaces. They show that network-flow techniques can be used to establish many of the known results in auction theory in a very elegant and easily interpretable manner. They also show how to simplify the associated optimization problem by identifying and relaxing provably redundant incentive compatibility constraints. In Vohra and Malakhov (2005), the authors use these techniques to identify the optimal mechanisms for an auction with capacitated bidders where both the capacity and marginal values are private information and the bidders are only allowed to lie about capacities in one direction. Thus, the model they consider is identical to the one discussed here and to an extent their work influences the results in this paper. The main methodological contributions that distinguishes our work are as follows.

(a) In Vohra and Malakhov (2005), the authors restrict attention to only those allocation rules that are monotone in the capacity dimension (i.e. the "special" type). We show that any allocation rule that is monotone in the marginal cost for a fixed capacity bid can be made incentive compatible by offering a side-payment to the suppliers that is only a function of the capacity bid (see Lemma 1). Thus, the space of all incentive compatible mechanisms is much larger than the one considered in Vohra and Malakhov (2005). Our characterization result also implies that the transfer payment is no longer uniquely determined by the allocation rule.

    A reverse auction with capacitated suppliers is special in that the objective does *not* explicitly depend on capacity bid – the capacity bid only controls the feasibility of an allocation rule. This special structure allows one to conclude that, when the prior distribution is regular, the optimal allocation rule is monotone in the capacity bid and, therefore, the optimal side payment can be set to zero, i.e. the solution in Vohra and Malakhov (2005) is indeed optimal.

    When the objective function explicitly depends on the "special" type, e.g. in bin packing with privately known weights or scheduling with privately known deadlines, one cannot "regularize" the prior distribution. Consequently, the mechanism design problem even with one-sided lying remains a hard problem.

(b) We develop a new ironing procedure which allows us to characterize the optimal mechanism under milder regularity conditions. See Section 3.2 for details.

(c) Vohra and Malakhov (2005) study *fixed* quantity auctions in discrete type space where all the bidders have a linear utility function. In contrast, we study variable quantity reverse auctions in a continuous type space. This allows us the flexibility of working with more general utility structures.

Notation

We denote vectors by boldface lowercase letters, e.g. $\mathbf{x}$. A vector indexed by $-i$, (for example $\mathbf{x}_{-i}$) denotes the vector $\mathbf{x}$ with the i-th component excluded. We use the convention $\mathbf{x} = (x_i, \mathbf{x}_{-i})$. Scalar (resp. vector) functions are denoted by lowercase (resp. boldface) letters, e.g. $x_i(\theta_i, \theta_{-i})$ (resp. $\mathbf{x}(\theta_i, \theta_{-i})$) and conditional expectation of functions by the uppercase of the same letter, e.g. $X_i(\theta_i) \equiv \mathbb{E}_{\theta_{-i}}\, x_i(\theta_i, \theta_{-i})$ (resp. $\mathbf{X}_i(\theta_i) = \mathbb{E}_{\theta_{-i}}[\mathbf{x}_i(\theta_i, \theta_{-i})]$. The possible misreport of the true parameters are represented with a hat over the same variable, e.g. $\hat{\theta}$.

2 Reverse auctions with finite supplier capacities

We consider a single period model with one buyer (retailer, manufacturer, etc.) and n suppliers. The buyer purchases a single commodity from the suppliers and resells it in the consumer market. The buyer receives an expected revenue, $R(q)$ from selling q units of the product in the consumer market – the expectation is over the random demand realization and any other randomness involved in the downstream market for the buyer that is not contractible. Thus, the side-payment to the suppliers cannot be contingent on the demand realization. We assume $R(q)$ is strictly concave with $R(0) = 0$, $R'(0) = \infty$ and $R'(\infty) = 0$, so that quantity ordered by the buyer is non-zero and bounded. Without this assumption the results in this paper would remain qualitatively the same; however, the optimal mechanism would have a reservation cost above which the buyer will not order anything. Characterizing the optimal reserve cost is straightforward and is well-studied (see, e.g. Dasgupta and Spulber, 1990).

Supplier i, $i = 1, \ldots, n$, has a constant marginal production cost $c_i \in [\underline{c}, \bar{c}] \subset (0, \infty)$ and finite capacity $q_i \in [\underline{q}, \bar{q}] \subset (0, \infty)$. The joint distribution function of marginal cost c_i and production capacity q_i is denoted by F_i. We assume that (c_i, q_i) and (c_j, q_j) are independently distributed when $i \neq j$, i.e. our model is an *independent private value* (IPV) model. We assume that distribution functions $\{F_i\}_{i=1}^n$ are common knowledge; however, the realization (c_i, q_i) is only known to supplier i. The buyer seeks a revenue maximizing procurement mechanism that ensures that all suppliers participate in the auction.

We employ the direct mechanism approach, i.e. the buyer asks suppliers to directly bid their private information (c_i, q_i). The revelation principle (see Myerson, 1981; Harris and Townsend, 1981) implies that for any given mechanism one can construct a direct mechanism that has the same point-wise allocation and transfer payment as the given mechanism. Since both mechanisms result in the same expected profit for the buyer, it follows that there is no loss of generality in restricting oneself to direct mechanisms.

We denote the true type of supplier by $\mathbf{b}_i = (c_i, q_i)$ and the supplier i bid by $\widehat{\mathbf{b}}_i = (\hat{c}_i, \hat{q}_i)$. Let $\mathbf{b} = (\mathbf{b}_1, ..., \mathbf{b}_n)$ and $\widehat{\mathbf{b}} = (\widehat{\mathbf{b}}_1, ..., \widehat{\mathbf{b}}_n)$. Let $\mathbf{B} \equiv \left([\underline{c}, \bar{c}] \times [\underline{q}, \bar{q}]\right)^n$ denote the type space. A procurement mechanism consists of

(a) an allocation function $\mathbf{x} : \mathbf{B} \to \mathbb{R}^n_+$ that for each bid vector $\widehat{\mathbf{b}}$ specifies the quantity to be ordered from each of the suppliers, and

(b) a transfer payment function $\mathbf{t} : \mathbf{B} \to \mathbb{R}^n$ that maps each bid vector $\widehat{\mathbf{b}}$ to the transfer payment from the buyer to the suppliers.

The buyer seeks an allocation function $\mathbf{x}$ and a transfer function $\mathbf{t}$ that maximizes the ex-ante expected profit

$$\Pi(\mathbf{x}, \mathbf{t}) \equiv \mathbb{E}_{\mathbf{b}} \left[R\left(\sum_{i=1}^n x_i(\mathbf{b}) \right) - \sum_{i=1}^n t_i(\mathbf{b}) \right]$$

subject to the following constraints.

1. feasibility: $x_i(\mathbf{b}) \leq q_i$ for all $i = 1, \ldots, n$, and $\mathbf{b} \in \mathbf{B}$,

2. *incentive compatibility* (**IC**): Conditional on their beliefs about the private information of other bidders, truthfully revealing their private information is weakly dominant for all suppliers, i.e.

$$(c_i, q_i) \in \operatorname*{argmax}_{\substack{\hat{c}_i \in [\underline{c}, \bar{c}] \\ \hat{q}_i \in [\underline{q}, q_i]}} \mathbb{E}_{\mathbf{b}_{-i}} \left\{ t_i((\hat{c}_i, \hat{q}_i), \mathbf{b}_{-i}) - c_i x_i((\hat{c}_i, \hat{q}_i), \mathbf{b}_{-i}) \right\}, \qquad i = 1, \ldots, n, \tag{1}$$

   The above definition of incentive compatibility is called Bayesian incentive compatibility (see Appendix **??**). Note that the range for the capacity bid $\hat{q}_i$ is $[\underline{q}, q_i]$, i.e. we do not allow the supplier to overbid capacity. This can be justified by assuming that the supplier incurs a heavy penalty for not being able to deliver the allocated quantity.

3. *individual rationality* (**IR**): The expected interim surplus of each supplier firm is non-negative, for all $i = 1, \ldots, n$, and $\mathbf{b} \in \mathbf{B}$, i.e.

$$\pi_i(\mathbf{b}_i) \equiv \mathbb{E}_{\mathbf{b}_{-i}} \left[t_i(\mathbf{b}) - c_i x_i(\mathbf{b}) \right] = T_i(c_i, q_i) - c_i X_i(c_i, q_i) \geq 0. \tag{2}$$

   Here we have assumed that the outside option available to the suppliers is constant and is normalized to zero.

For any procurement mechanism $(\mathbf{x}, \mathbf{t})$, the *offered* expected surplus $\rho_i(\hat{c}_i, \hat{q}_i)$ when supplier i bids $(\hat{c}_i, \hat{q}_i)$ is defined as follows

$$\rho_i(\hat{c}_i, \hat{q}_i) = T_i(\hat{c}_i, \hat{q}_i) - \hat{c}_i X_i(\hat{c}_i, \hat{q}_i)$$

The offered surplus is simply a convenient way of expressing the expected transfer payment. The expected surplus $\pi_i(c_i, q_i)$ of supplier i with true type (c_i, q_i) when she bids $(\hat{c}_i, \hat{q}_i)$ is given by

$$\pi_i(c_i, q_i) = T_i(\hat{c}_i, \hat{q}_i) - c_i X_i(\hat{c}_i, \hat{q}_i) = \rho_i(\hat{c}_i, \hat{q}_i) + (\hat{c}_i - c_i) X_i(\hat{c}_i, \hat{q}_i).$$

The true surplus π_i equals the offered surplus ρ_i if the mechanism $(\mathbf{x}, \mathbf{t})$ is **IC**.

To further motivate the procurement mechanism design problem, we elaborate on a supplier's incentives to lie about capacity and then consider some illustrative special cases.

2.1 Incentive to underbid capacity

In this section we show that auctions that ignore the capacity information are not incentive compatible. In particular, the suppliers have an incentive to underbid capacity.

Suppose we ignore the private capacity information and implement the classic K^{th} price auction where the marginal payment to the supplier is equal to the cost of the first losing supplier, i.e. lowest cost supplier among those that did not receive any allocation. Then truthfully bidding the marginal cost is a dominant strategy. However, we show below that in this mechanism the suppliers have an incentive to underbid capacity. Underbidding creates a fake shortage resulting in an increase in the transfer payment that can often more than compensates the loss due to a decrease in the allocation. The following example illustrates these incentives in dominant strategy and Bayesian framework.

Example 1. Consider a procurement auction with three capacitated suppliers implemented as the K^{th} price auction. Let $\underline{c} = 1, \bar{c} = 5, \underline{q} = .01$ and $\bar{q} = 6$. Suppose the capacity realization is $(q_1, q_2, q_3) = (5, 1, 5)$ and the marginal cost realization is $(c_1, c_2, c_3) = (1, 1, 5)$. Suppose the buyer wants to procure 5 units and that the spot price, i.e. the outside publicly known cost at which the buyer can procure unlimited quantity is equal to 10. (We need to have an outside market when modeling fixed quantity auction because the realized total capacity of the suppliers can be less than the fixed quantity that needs to be procured.)

Assume that suppliers 2 and 3 bid truthfully. Consider supplier 1. If she truthfully reveals her capacity, her surplus is \$0; however, if she bids $\hat{q}_1 = 4 - \epsilon$, her surplus is equal to $\$9(4 - \epsilon)$. Thus, bidding truthfully is not a dominant strategy for supplier 1.

Next, we show that for appropriately chosen asymmetric prior distributions supplier 1 has incentives to underbid capacity even in the Bayesian framework. Assume that the marginal cost and capacity are independently distributed. Let $(c_1, q_1) = (1, 5)$. Thus $\mathbb{P}((1 \leq c_2) \cap (1 \leq c_3)) = 1$. Let the capacity distribution F_i^q, $i = 2, 3$, be such that $\mathbb{P}(q_2 + q_3 \leq 1) > 1 - \epsilon$ for some $0 < \epsilon \ll 1$. Then the expected surplus $\pi_1(1, 5)$, if supplier 1 bids her capacity truthfully, is upper bounded by $5 \times (\bar{c} - 1) = 20$. On the other hand the expected surplus if she bids $4 - \epsilon$ is lower bounded by $9 \times (4 - \epsilon) \times (1 - \epsilon)$. Thus, supplier 1 has ex-ante incentive to underbid capacity. ■

Figure 1 shows two uniform price auction mechanisms, the K^{th} price auction and the market clearing mechanism. In our model, the suppliers can change the supply ladder curve both in terms of location of the jumps (by misreporting costs) and the magnitude of the jump (by misreporting capacity). We know that in a model with commonly known capacities, the fixed quantity optimal auction can be implemented as K^{th} price auction. We showed in the example above that in the K^{th} price auction with privately known capacity, the suppliers can "*game*" the mechanism.

This effect is also true if prices are determined by the market clearing condition. Suppose the suppliers truthfully reveal their marginal costs and the buyer aggregates these bids to form the supply curve $Q(p) = \sum_{i=1}^{n} \hat{q}_i \mathbf{1}_{\{c_i \leq p\}}$. The demand curve $D(p)$ in this context is given by

$$D(p) = \operatorname*{argmax}_{u \geq 0} \left[R(u) - pu\right] = (R')^{-1}(p).$$

Thus, the equilibrium price p^* is given by the solution of the market clearing condition $(R')^{-1}(p) = Q(p^*)$ (see Figure 1). The model primitives ensure that the market clearing price $p^* \in (0, \infty)$.

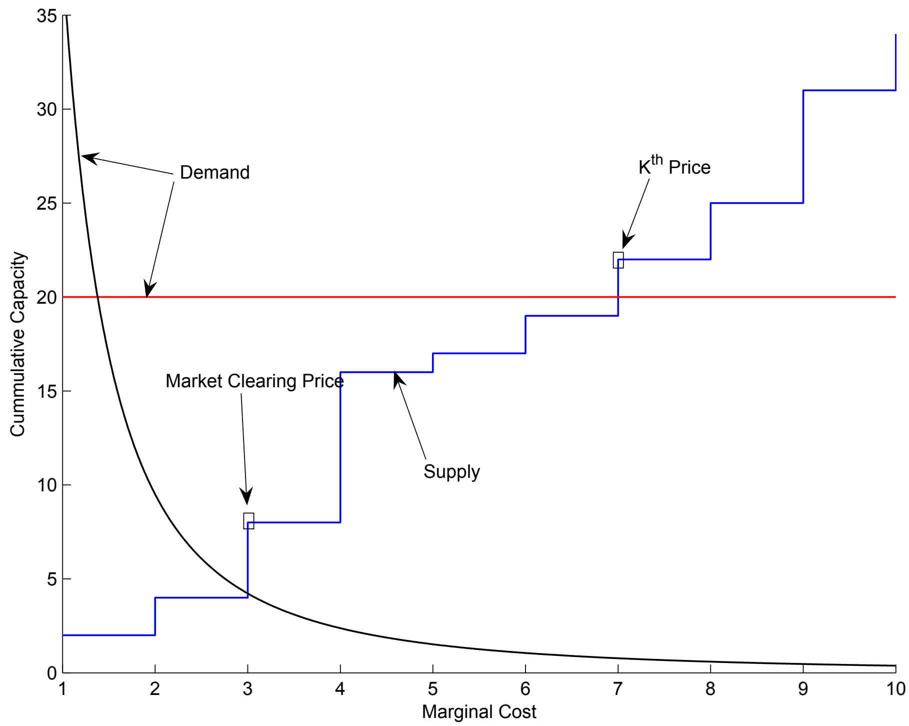

Figure 1: Uniform price auctions: K^{th} price auction and market clearing price auction

In such a setting, as in the K-th price auction, the supplier with low cost and high capacity can at times increase surplus by underbidding capacity because the increase in the marginal (market clearing) price can offset the decrease in allocation.

The above discussion shows that both the K-th price auction and the market-clearing mechanism are not truth revealing. In § 2.2.3 we show that if the suppliers bid the cost truthfully for exogenous reasons, the buyer can extract all the surplus, i.e. the buyer does not pay any information rent to the suppliers for the capacity information. In this mechanism the transfer payments are simply the true costs of the supplier and the quantity allocated is a monotonically decreasing function of the marginal cost. This optimal mechanism is *discriminatory* and unique. In particular, with privately known capacities, there does not exist a *uniform price* optimal auction. Ausubel (2004) shows that a modified market clearing mechanism, where items are awarded at the price that they are "clinched", is efficient, i.e. socially optimal ((see, also Ausubel and Cramton, 2002)).

2.2 Relaxations

In this section we discuss some special cases of the procurement mechanism design problem formulated in § 2.

2.2.1 Full information (or first-best) solution

Suppose all suppliers bid truthfully. It is clear that in this setting the surplus of each supplier would be identically zero. Denote the marginal cost of supplier firm with i^{th} lowest marginal cost by $c_{[i]}$ and it's capacity by $q_{[i]}$. Then the piece-wise convex linear cost function faced by the buyer is given by

$$c(x) = \sum_{j=1}^{i-1} q_{[j]} c_{[i]} + \left(x - \sum_{j=1}^{i-1} q_{[j]} \right) c_{[i]} \quad \text{for } \sum_{j=1}^{i-1} q_{[j]} \leq x \leq \sum_{j=1}^{i} q_{[j]} \tag{3}$$

The optimal procurement strategy for the buyer is the same as that of a buyer facing a single supplier with piece wise linear convex production cost $c(x)$. Clearly, multi-sourcing is optimal with a number of lowest cost suppliers producing at capacity and at most one supplier producing below capacity.

Multiple sourcing can also occur in an uncapacitated model when the production costs are nonlinear. We expect that a risk averse buyer would also find it advantageous to multi-source to diversify the ex-ante risk due to the asymmetric information. Since, to the best of our knowledge, the problem of optimal auctions with a risk averse principal has not been fully explored in the literature, this remains a conjecture.

2.2.2 Second-degree price discrimination with a single capacitated supplier

Suppose there is a single supplier with privately known marginal cost and capacity. Suppose the capacity and cost are independently distributed. Let $F(c)$ and $f(c)$ denote, respectively, the cumulative distribution function (CDF) and density of the marginal cost c and suppose the hazard rate $\frac{f(c)}{F(c)}$ is monotonically decreasing, i.e. we are in the so-called regular care (Myerson, 1981). Note that this setting is the procurement counterpart of second degree price discrimination in the monopoly pricing model.

We will first review the optimal mechanism when the supplier is uncapacitated. Using the indirect utility approach, the buyer's problem can formulated as follows.

$$\max_{\substack{x(\cdot) \geq 0 \\ x(\cdot) \text{monotone}}} \quad \mathbb{E}_c \left[R(x(c)) - \left(c + \frac{F(c)}{f(c)} \right) x(c) \right]. \tag{4}$$

Let $x^*(c)$ denote the optimal solution of the relaxation of (4) where one ignores the monotonicity assumption, i.e.

$$x^*(c) \in \underset{x \geq 0}{\operatorname{argmax}} \left\{ R(x) - \left(c + \frac{F(c)}{f(c)} \right) x \right\}.$$

Then, regularity implies that x^* is a monotone function of c, and is, therefore, feasible for (4). The transfer payment $t^*(c)$ that makes the optimal allocation x^* incentive compatible is given by

$$t^*(c) = cx^*(c) + \int_c^{\bar{c}} x^*(u) du.$$

Since the optimal allocation $x^*(c)$ and the transfer payment $t^*(c)$ are both monotone in c, the cost parameter c can be eliminated to obtain the transfer t directly in terms of the allocation x, i.e.

a *tariff* $t^*(x)$. The indirect tariff implementation is very appealing for implementation as it can "posted" and the supplers can simply self-select the production quantity based on the posted tariff.

Now consider the case of a capacitated supplier. Feasibility requires that for all $c \in [\underline{c}, \bar{c}]$, $0 \le x(c) \le q$. Suppose the supplier bids the capacity truthfully. (We justify this assumption below.) Then the buyer's problem is given by

$$\max_{\substack{x(\cdot,\cdot)\ge 0 \\ x(\cdot,q)\text{monotone}}} \mathbb{E}_{(c,q)}\left[R(x(c,q)) - \Big(c + \frac{F(c)}{f(c)}\Big)x(c,q)\right] \tag{5}$$

where F denotes the marginal distribution of the cost. Set the allocation $\hat{x}(c,q) = \min\{x^*(c), q\}$, where x^* denotes the optimal solution of the uncapacitated problem (4). Then $\hat{x}$ is clearly feasible for (5). Moreover,

$$\hat{x}(c,q) \in \operatorname*{argmax}_{0\le x\le q}\left\{R(x) - \Big(c + \frac{F(c)}{f(c)}\Big)x\right\}.$$

Thus, $\hat{x}$ is an optimal solution of (5). As before, set transfer payment $\hat{t}(c,q) = c\hat{x}(c,q) + \int_c^{\bar{c}} \hat{x}(u,q)du$. Then, the supplier surplus in the solution $(\hat{x}, \hat{t})$ is non-decreasing in the capacity bid q. Therefore, it is weakly dominant for the supplier to bid the capacity truthfully, and our initial assumption is justified. Note that the supplier surplus $\hat{\pi}(c,q) = \int_c^{\bar{c}} \hat{x}(u,q)du$.

The fact that the capacitated solution $\hat{x}(c,q) = \min\{x^*(c), q\}$ is simply a truncation of the uncapacitated solution $x^*(c)$ allows one to implement it in a very simple manner. Suppose the buyer offers the seller the tariff $t^*(x)$ corresponding to the uncapacitated solution. Then the solution $\tilde{x}$ of the seller's optimization problem $\max_{0\le x\le q}\{t^*(x) - cx\}$ is given by

$$\tilde{x} = \min\{x^*(c), q\} = \hat{x}(c,q),$$

i.e. the quantity supplied is the same as that dictated by the optimal capacitated mechanism.

Define $c_q = \sup\{c \in [\underline{c}, \bar{c}] : x^*(c) \ge q\}$. Then the monotonicity of $x^*(c)$ implies that

$$\tilde{x} = \hat{x}(c,q) = \begin{cases} x^*(c), & c > c_q, \\ q, & c \le c_q. \end{cases}$$

Then, for all $c > c_q$, the supplier requests $x^*(c)$ and receives a surplus

$$\tilde{\pi}(c) = t^*(x^*(c)) - cx^*(c) = \int_c^{\bar{c}} x^*(u)du = \int_c^{\bar{c}} \min\{x^*(u), q\}du = \int_c^{\bar{c}} \hat{x}(u,q)du = \hat{\pi}(c,q).$$

For $c \le c_q$, the supplier request q and the surplus

$$\begin{aligned} \tilde{\pi}(c) &= t^*(q) - cq, \\ &= t^*(x^*(c_q)) - c_q q + (c_q - c)q, \\ &= \pi^*(c_q) + (c_q - c)q = \int_{c_q}^{\bar{c}} x^*(u)du + \int_c^{c_q} q du = \int_c^{\bar{c}} \hat{x}(u,q)du = \hat{\pi}(c,q). \end{aligned}$$

Thus, the supplier surplus in the tariff implementation is $\hat{\pi}(c,q)$, the surplus associated with optimal capacitated mechanism. Consequently, it follows that the "full" tariff implements the capacitated optimal mechanism! This immediately implies that the buyer does need to know the capacity of the supplier, and pays zero information rent for the capacity information. In the next section we show that the assumption of independence of capacity and cost is critical for this result.

2.2.3 Marginal Cost common knowledge

Suppose the marginal costs are common knowledge and only the production capacities are privately known. Then the optimal procurement mechanism maximizes

$$\max_{(\mathbf{x},\mathbf{t})} \mathbb{E}_q \left[R\Big(q_i \sum_{i=1}^{n} x_i(q)\Big) - \sum_{i=1}^{n} t_i(q) \right]$$

such that the expected supplier i surplus $T_i(q_i) - c_i X_i(q_i)$ is weakly increasing in q_i (**IC**) and nonnegative (**IR**).

Not surprisingly, the first-best or the full-information solution works in this case. Set the transfer payment equal to the production costs of the supplier, i.e. $t_i(q) = c_i x_i(q)$. Then the supplier surplus is zero and the buyer's optimization problem reduces to the full-information case. Since the full-information allocation $x_i(\hat{q}_i, q_{-i})$ is weakly increasing in $\hat{q}_i$ for all q_{-i}, bidding the true capacity is a weakly dominant strategy for the suppliers. Thus, the buyer can effectively ignore the **IC** constraints above and follow the full information allocation scheme and extract all the supplier surplus. The fact that, conditional on knowing the cost, the buyer does not offer any informational rent for the capacity information is crucial to the result in the next section.

3 Characterizing Optimal Direct Mechanism

We use the standard indirect utility approach to characterize all incentive compatible and individually rational direct mechanisms and the minimal transfer payment function that implements a given incentive compatible allocation rule (see Lemma 1). The characterization of the transfer payment allows us to write the expected profit of the buyer for a given incentive compatible allocation rule as a function of the allocation rule and the offered surplus $\rho_i(\bar{c}, q)$ (see Theorem 1). To proceed further, we make the following assumption.

Assumption 1. *For all $i = 1, 2, \cdots, n$, the joint density $f_i(c_i, q_i)$ has full support. Therefore, the conditional density $f_i(c_i|q_i)$ also has full support.*

Lemma 1. *Procurement mechanisms with capacitated suppliers satisfy the following.*

(a) A feasible allocation rule $\mathbf{x} : \mathbf{B} \to \mathbb{R}_+^n$ is **IC** *if, and only if, the expected allocation $X_i(c_i, q_i)$ is non-increasing in the cost parameter c_i.*

(b) A mechanism $(\mathbf{x}, \mathbf{t})$ is **IC** *and* **IR** *if, and only if, the allocation rule $\mathbf{x}$ satisfies (a) and the offered surplus $\rho_i(\hat{c}_i, \hat{q}_i)$ when supplier i bids $(\hat{c}_i, \hat{q}_i)$ is of the form*

$$\rho_i(\hat{c}_i, \hat{q}_i) = \rho_i(\bar{c}, \hat{q}_i) + \int_{\hat{c}_i}^{\bar{c}} X_i(u, \hat{q}_i) du \tag{6}$$

with $\rho_i(\hat{c}_i, \hat{q}_i)$ non-negative and non-decreasing in $\hat{q}_i$ for all $\hat{c}_i \in [\underline{c}, \bar{c}]$.

Remark 1. *Recall that the offered surplus ρ_i is, in fact, equal to the surplus π_i when the allocation rule $\mathbf{x}$ (and the associated transfer payment $\mathbf{t}$) is* **IC**.

Proof: Fix the mechanism $(\mathbf{x}, \mathbf{t})$. Then the supplier i expected surplus $\pi_i(c_i, q_i)$ is given by

$$\pi_i(c_i, q_i) = \max_{\substack{\hat{c}_i \in [\underline{c}, \bar{c}] \\ \hat{q}_i \in [\underline{q}, q_i]}} \left\{ T_i(\hat{c}_i, \hat{q}_i) - c_i X_i(\hat{c}_i, \hat{q}_i) \right\}. \tag{7}$$

Note that the capacity bid $\hat{q}_i \leq q_i$, the true capacity. This plays an important role in the proof. From (7), it follows that for all fixed $q \in [\underline{q}, \bar{q}]$, the surplus $\pi_i(c_i, q_i)$ is convex in the cost parameter c_i. (There is, however, no guarantee that $\pi_i(c_i, q_i)$ is jointly convex in (c_i, q_i).) Consequently, for all fixed $q \in [\underline{q}, \bar{q}]$, the function $\pi_i(c_i, q_i)$ is absolutely continuous in c and differentiable almost everywhere in c.

Since x is **IC**, it follows that (c_i, q_i) achieves the maximum in (7). Thus, in particular,

$$c_i \in \underset{\hat{c}_i \in [\underline{c}, \bar{c}]}{\operatorname{argmax}} \left\{ T_i(\hat{c}_i, q_i) - c_i X_i(\hat{c}_i, q_i) \right\}, \tag{8}$$

i.e. if supplier i bids capacity q truthfully, it is still optimal for her to bid the cost truthfully. Since $\pi_i(c_i, q_i)$ is convex in c_i, (8) implies that

$$\frac{\partial \pi_i(c, q)}{\partial c} = -X_i(c, q), \quad \text{a.e.} \tag{9}$$

Consequently, $X_i(c, q)$ is non-increasing in c for all $q \in [\underline{q}, \bar{q}]$. This proves the forward direction of the assertion in part (a).

To prove the converse of part (a), suppose $X_i(c_i, q_i)$ is non-increasing in c_i for all q_i. Set the offered surplus

$$\rho_i(\hat{c}_i, \hat{q}_i) = \bar{\rho}_i(\hat{q}_i) + \int_c^{\bar{c}} X_i(u, \hat{q}_i) du$$

where the function $\bar{\rho}_i(\hat{q}_i)$ so that $\rho_i(\hat{c}_i, \hat{q}_i)$ is non-decreasing in $\hat{q}_i$ for all $\hat{c}_i \in [\underline{c}, \bar{c}]$. There are many feasible choices for $\bar{\rho}(\hat{q}_i)$. In particular, if $\frac{\partial X_i(c,q)}{\partial q}$ exists a.e., one can set,

$$\bar{\rho}_i(\hat{q}_i) = \sup_{c_i \in [\underline{c}, \bar{c}]} \Big\{ \int_{\underline{q}}^{q_i} \int_{c_i}^{\bar{c}} \Big(\frac{\partial X_i(t, z)}{\partial z} \Big)^{-} dt dz \Big\}.$$

For any such choice of $\bar{\rho}_i$, the supplier i surplus

$$\begin{aligned}
\pi_i(\hat{c}_i, \hat{q}_i) &= \rho_i(\hat{c}_i, \hat{q}_i) + (\hat{c}_i - c_i) X_i(\hat{c}_i, \hat{q}_i), \\
&= \bar{\rho}_i(\hat{q}_i) + \int_{\hat{c}_i}^{\bar{c}} X_i(u, \hat{q}_i) du + (\hat{c}_i - c_i) X_i(\hat{c}_i, \hat{q}_i), \\
&= \bar{\rho}_i(\hat{q}_i) + \int_{c_i}^{\bar{c}_i} X_i(u, \hat{q}_i) du + \int_{\hat{c}_i}^{c_i} X_i(u, \hat{q}_i) du + (\hat{c}_i - c_i) X_i(\hat{c}_i, \hat{q}_i), \\
&\leq \bar{\rho}_i(\hat{q}_i) + \int_{c_i}^{\bar{c}} X_i(u, \hat{q}_i) du, \qquad (10) \\
&\leq \bar{\rho}_i(q_i) + \int_{c_i}^{\bar{c}} X_i(u, q_i) du, \qquad (11) \\
&= T_i(c_i, q_i) - c_i X_i(c_i, q_i) = \pi_i(c_i, q_i),
\end{aligned}$$

where (10) follows from the fact that $X_i(c,q)$ in non-increasing in c for all fixed q and (11) follows from the $\rho_i(\hat{c}_i, \hat{q}_i)$ is non-decreasing in $\hat{q}_i$ and $\hat{q}_i \leq q_i$. Thus, we have established that it is weakly dominant for supplier i to bid truthfully, or equivalently x is an incentive compatible allocation.

¿From (9) we have that whenever x is **IC** we must have that the supplier surplus is of the form

$$\pi_i(c_i, q_i) = \pi_i(\bar{c}, q_i) + \int_c^{\bar{c}} X_i(u, q_i)du.$$

Since x is **IR**, $\pi_i(\bar{c}_i, q_i) \geq 0$, and, since x is **IC**,

$$q_i \in \underset{\hat{q}_i \leq q_i}{\operatorname{argmax}} \{T_i(c_i, \hat{q}_i) - c_i X_i(c_i, \hat{q}_i)\} = \underset{\hat{q}_i \leq q_i}{\operatorname{argmax}} \{\pi_i(c_i, \hat{q}_i)\}.$$

Thus, we must have that $\pi_i(c_i, q_i)$ is non-decreasing in q_i for all $c_i \in [\underline{c}, \bar{c}]$. This establishes the forward direction of part (b).

Suppose the offered surplus if of the form (6) then $(\mathbf{x}, \mathbf{t})$ satisfies **IR**. Since $X_i(c_i, q_i)$ is non-increasing in c_i for all q_i, it follows that $\pi_i(c_i, q_i)$ is convex in c_i for all q_i and $\frac{\partial \pi_i(c_i,q_i)}{\partial c_i} = -X_i(c_i, q_i)$. Consequently,

$$\pi_i(\hat{c}_i, \hat{q}_i) = \rho_i(\hat{c}_i, \hat{q}_i) + (c_i - \hat{c}_i)\big(- X_i(\hat{c}_i, \hat{q}_i)\big) \leq \pi_i(c_i, \hat{q}_i) \leq \pi_i(c_i, q_i),$$

where the last inequality follows from the fact that $\pi_i(c_i, q_i)$ is non-decreasing in q_i for all c_i and $\hat{q}_i \leq q_i$. Thus, we have establishes that $(\mathbf{x}, \mathbf{t})$ is **IC**. ■

Next, we use the results in Lemma 1 to characterize the buyer's expected profit.

Theorem 1. *Suppose Assumption 1 holds. Then the buyer profit $\Pi(\mathbf{x}, \mathbf{t})$ corresponding to any feasible allocation rule $\mathbf{x} : \mathbf{B} \to \mathbb{R}^n_+$ that satisfies* **IC** *and* **IR** *is given by*

$$\Pi(\mathbf{x}, \bar{\boldsymbol{\rho}}) = \mathbb{E}_b \left[R\left(\sum_{i=1}^n x_i(b)\right) - \sum_{i=1}^n x_i(b) H_i(c_i, q_i) - \sum_{i=1}^n \bar{\rho}_i(q_i) \right], \tag{12}$$

where $\bar{\rho}_i(q_i)$ is the surplus offered when the supplier i bid is $(\bar{c}, q_i)$ and $H_i(c,q)$ denotes the virtual cost defined in Assumption 2.

Remark 2. *Theorem 1 implies that the buyer's profit is determined by* both *the allocation rule* $\mathbf{x}$ *and offered surplus $\bar{\boldsymbol{\rho}}(q)$ when supplier i bid is $(\bar{c}, q)$. We emphasize this by denoting the buyer profit by $\Pi(\mathbf{x}, \bar{\boldsymbol{\rho}})$.*

Proof: From Lemma 1, we have that the offered supplier i surplus $\rho_i(c_i, q_i)$ under any **IC** and **IR** allocation rule x is of the form

$$\rho_i(c_i, q_i) = \rho_i(\bar{c}, q_i) + \int_{c_i}^{\bar{c}} X_i(t, q_i)dt$$

Thus, the buyer profit function is

$$\Pi = \mathbb{E}_{\mathbf{b}} \left[R\left(\sum_{i=1}^n x_i(\mathbf{b})\right) - \sum_{i=1}^n \big(c_i x_i(\mathbf{b}) + \rho_i(\bar{c}, q_i)\big) \right] - \sum_{i=1}^n \left(\int_{\underline{q}}^{\bar{q}} \int_{\underline{c}}^{\bar{c}} \int_{c_i}^{\bar{c}} X_i(u_i, q_i) du_i f_i(c_i, q_i) dc_i dq_i \right).$$

By interchanging the order of integration, we have

$$\int_{\underline{c}}^{\bar{c}} dc_i f_i(c_i, q_i) \int_{c_i}^{\bar{c}} du_i X_i(u_i, q_i) = \int_{\underline{c}}^{\bar{c}} du_i X_i(u_i, q_i) \int_{\underline{c}}^{t} dc f_i(c, q_i) = \int_{\underline{c}}^{\bar{c}} X_i(c_i, q_i) F_i(c_i \mid q_i) f_i(q_i) dc_i.$$

Substituting this back into the expression for profit, we get

$$\begin{aligned}
\Pi(x) &= \mathbb{E}_{\mathbf{b}}\left[R\left(\sum_{i=1}^n x_i(\mathbf{b})\right) - \sum_{i=1}^n \left(c_i x_i(\mathbf{b}) + \rho_i(\bar{c}, q_i)\right)\right] - \sum_{i=1}^n \left(\int_{\underline{q}}^{\bar{q}} \int_{\underline{c}}^{\bar{c}} X_i(c_i, q_i) F_i(c_i|q_i) f_i(q_i) dc_i dq_i\right), \\
&= \mathbb{E}_{\mathbf{b}}\left[R\left(\sum_{i=1}^n x_i(\mathbf{b})\right) - \sum_{i=1}^n \left(c_i x_i(\mathbf{b}) + \rho_i(\bar{c}, q_i)\right)\right] - \sum_{i=1}^n \mathbb{E}_{\mathbf{b}}\left[x_i(\mathbf{b}) \frac{F_i(c_i|q_i)}{f_i(c_i|q_i)}\right] \\
&= \mathbb{E}_{\mathbf{b}}\left[R\left(\sum_{i=1}^n x_i(\mathbf{b})\right) - \sum_{i=1}^n \left(c_i + \frac{F_i(c_i|q_i)}{f_i(c_i|q_i)}\right) x_i(\mathbf{b}) - \sum_{i=1}^n \rho_i(\bar{c}, q_i)\right].
\end{aligned}$$

This establishes the result. ■

The virtual marginal costs $H_i(c, q)$ in our model are very similar to the virtual marginal costs in the uncapacitated reverse auction model; except that the virtual costs are now defined in terms of the distribution of the marginal cost c_i *conditioned* on the capacity bid q_i. Thus, the capacity bid provides information only if the cost and capacity are correlated. (See § 2.2.2 and § 2.2.3 for more on this issue). Next, we characterize the optimal allocation rule under the regularity Assumption 2 and to a limited extent under general model primitives.

3.1 Optimal mechanism in the regular case

In this section, we make the following additional assumption.

Assumption 2 (Regularity)**.** *For all $i = 1, 2, \cdots, n$, the virtual cost function $H_i(c_i, q_i) \equiv c_i + \frac{F_i(c_i|q_i)}{f_i(c_i|q_i)}$ is non-decreasing in c_i and non-increasing in q_i.*

Assumption 2 is called the *regularity* condition. It is satisfied when the conditional density of the marginal cost given capacity, is log concave in c_i, and the production cost and capacity are, loosely speaking, "negatively affiliated" in such a way that $\frac{F_i(c_i|q_i)}{f_i(c_i|q_i)}$ is nondecreasing in q_i. This is true, for example, when the cost and capacity are independent.

For $\mathbf{b} \in \mathbf{B}$, define

$$\mathbf{x}^*(\mathbf{b}) \equiv \operatorname*{argmax}_{0 \leq \mathbf{x} \leq \mathbf{q}} \left\{ R\left(\sum_{i=1}^n x_i\right) - \sum_{i=1}^n x_i H_i(c_i, q_i) \right\}, \tag{13}$$

where the inequality $0 \leq \mathbf{x} \leq \mathbf{q}$ is interpreted component-wise. We call $\mathbf{x}^* : \mathbf{B} \to \mathbb{R}_+^n$ the point-wise optimal allocation rule. Since (13) is identical to the full information problem with the cost c_i replaced by the *virtual cost* $H_i(c_i, q_i)$, it follows that (13) can be solved by aggregating all the suppliers into one meta-supplier. Denote the virtual cost of supplier with i^{th} lowest virtual cost by $h_{[i]}$ and the corresponding capacity by $q_{[i]}$. Then the buyer faces a piece-wise convex linear cost function $h(q)$ given by

$$h(q) = \sum_{j=1}^{i-1} q_{[j]} h_{[i]} + \left(q - \sum_{j=1}^{i-1} q_{[j]}\right) c_{[i]}, \tag{14}$$

for $\sum_{j=1}^{i-1} q_{[j]} \le q \le \sum_{j=1}^{i} q_{[j]}$, $i = 1, \ldots, n$, where $\sum_{j=1}^{0} q_{[j]}$ is set to zero. From the structure of the supply curve it follows that the optimal solution of (13) is of the form

$$x^*_{[i]} = \begin{cases} q_{[i]}, & [i] < [i]^*, \\ \le q_{[i]}, & [i] = [i]^*, \\ 0, & \text{otherwise}, \end{cases} \tag{15}$$

where $1 \le [i]^* \le n$.

Lemma 2. *Suppose Assumption 2 holds. Let $\mathbf{x}^* : \mathbf{B} \to \mathbb{R}^n_+$ denote the point-wise optimal defined in (13).*

(a) $x^*_i((c_i, q_i), \mathbf{b}_{-i})$ is non-increasing in c_i for all fixed q_i and $\mathbf{b}_{-i}$. Consequently, $X_i(c_i, q_i)$ is non-increasing in c_i for all q_i.

(b) $x^*_i((c_i, q_i), \mathbf{b}_{-i})$ is non-decreasing in q_i for all fixed c_i and $\mathbf{b}_{-i}$. Therefore, $X_i(c_i, q_i)$ is non-decreasing in q_i for all fixed c_i.

Proof: From (15) it is clear that $\mathbf{x}^*((c_i, q_i), \mathbf{b}_{-i})$ is non-increasing in the virtual cost $H_i(c_i, q_i)$. When Assumption 2 holds, the virtual cost $H_i(c_i, q_i)$ is non-decreasing in c_i for fixed q_i; consequently, the allocation x^*_i is non-increasing in the capacity bid q_i for fixed c_i and $\mathbf{b}_{-i}$. Part (a) is established by taking expectations of $\mathbf{b}_{-i}$. A similar argument proves (b). ■

We are now in position to prove the main result of this section.

Theorem 2. *Suppose Assumption 1 and 2 hold. Let $\mathbf{x}^*$ denote the point-wise optimal solution defined in (13). For $i = 1, \ldots, n$, set the transfer payment*

$$t^*_i(\widehat{\mathbf{b}}) = \hat{c}_i X^*_i(c_i, q_i) + \int_{\hat{c}_i}^{\bar{c}} X^*_i((u, \hat{q}_i))du. \tag{16}$$

Then $(\mathbf{x}^*, \mathbf{t}^*)$ is Bayesian incentive compatible revenue maximizing procurement mechanism.

Proof: From (12), we have that the buyer profit

$$\Pi(\mathbf{x}, \bar{\boldsymbol{\rho}}) \le \mathbb{E}_{\mathbf{b}} \left[\max_{\mathbf{0} \le \mathbf{x} \le \mathbf{q}} \left\{ R\left(\sum_{i=1}^{n} x_i \right) - \sum_{i=1}^{n} x_i H_i(c_i, q_i) \right\} \right] = \Pi(\mathbf{x}^*, \mathbf{0}).$$

Thus, all that remains to be shown is that the offered surplus ρ^*_i corresponding to the transfer payment $\mathbf{t}^*$ satisfies $\bar{\rho}^*_i(q_i) = \rho^*_i(\bar{c}_i, q_i) \equiv 0$, and $(\mathbf{x}^*, \mathbf{t}^*)$ is **IC** and **IR**.

From (16), it follows that the offered surplus

$$\rho^*_i(\hat{c}_i, \hat{q}_i) = \int_{\hat{c}_i}^{\bar{c}} X^*_i(u, \hat{q}_i)du. \tag{17}$$

Thus, $\bar{\rho}^*_i(q_i) = \rho^*_i(\bar{c}, q_i) \equiv 0$.

Next, Lemma 2 (a) implies that $X^*_i((\hat{c}_i, \hat{q}_i), \mathbf{b}_{-i})$ is non-increasing in c_i for all q_i. From Lemma 2 (b), we have that $X_i(u, \hat{q}_i)$ is non-decreasing in $\hat{q}_i$. From (17), it follows that $\pi_i(c_i, q_i)$ is non-decreasing in q_i for all c_i. Now, Lemma 1 (b) allows us to conclude that $(\mathbf{x}^*, \mathbf{t}^*)$ is **IC**.

Since $(\mathbf{x}^*, \mathbf{t}^*)$ satisfies **IC**, the offered surplus $\rho^*_i(c_i, q_i)$ is, indeed, the supplier surplus. Then (17) implies that $(\mathbf{x}^*, \mathbf{t}^*)$ is **IR**. ■

Next, we illustrate the optimal reverse auction on a simple example.

Example 2. Consider a procurement auction with two identical suppliers. Suppose the marginal cost c_i and capacity q_i of each of the suppliers are uniformly distributed over the unit square,

$$f_i(c_i, q_i) = 1 \quad \forall (c_i, q_i) \in [0,1]^2, i = 1, 2.$$

Therefore, the virtual costs

$$H_i(c_i, q_i) = c_i + \frac{F_i(c_i|q_i)}{f_i(c_i|q_i)} = c_i + c_i = 2c_i \quad \forall c_i \in [0,1], i = 1, 2.$$

It is clear that this example satisfies Assumption 1 and Assumption 2.

Suppose the buyer revenue function $R(q) = 4\sqrt{q}$. Then, it follows that buyer's optimization problem reduces to the point-wise problem

$$\mathbf{x}^*(\mathbf{c}, \mathbf{q}) = \underset{\mathbf{x} \le \mathbf{q}}{\operatorname{argmax}} \left\{ 4\sqrt{\sum_{i=1}^{2} x_i} - 2\sum_{i=1}^{2} c_i x_i \right\}.$$

The above constrained problem can be easily solved using the Karush-Kuhn-Tucker (KKT) conditions which are sufficient because of strict concavity of the buyer's profit function. For $i = 1, 2$, the solution is given by,

$$x_i^*(c, q) = \begin{cases} \frac{1}{c_i^2} & c_i \le c_{-i},\ q_i \ge \frac{1}{c_i^2}, \\ q_i & c_i \le c_{-i},\ q_i < \frac{1}{c_i^2}, \\ 0 & c_i \ge c_{-i},\ q_{-i} \ge \frac{1}{c_{-i}^2}, \\ \min\left\{\max\left\{0, \frac{1}{c_i^2} - q_{-i}\right\}, q_i\right\} & \text{otherwise.} \end{cases}$$

where $-i$, is the index of the supplier competing with supplier i. The corresponding expected transfer payments are given by equation (16). ■

In order for an allocation rule $\mathbf{x}$ to be Bayesian incentive compatible it is sufficient that the expected allocation $X_i(c_i, q_i)$ be weakly monotone in c_i and q_i. Assumption 2 ensures that the point-wise optimal allocation x_i^* is weakly monotone in c_i and q_i. This stronger property of $\mathbf{x}^*$ can be exploited to show that $\mathbf{x}^*$ can be implemented in the dominant strategy solution concept, i.e. there exist a transfer payment function under which truth telling forms an dominant strategy equilibrium.

Theorem 3. *Suppose Assumption 1 and Assumption 2 hold. For $i = 1, \dots, n$, let the transfer payment be*

$$t_i^{**}(\widehat{\mathbf{b}}) = \hat{c}_i x_i^*(\widehat{\mathbf{b}}) + \int_{\hat{c}_i}^{\bar{c}} x_i^*((u, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) du. \tag{18}$$

Then, $(\mathbf{x}^*, \mathbf{t}^{**})$ is an dominant strategy incentive compatible individually rational revenue maximizing procurement mechanism.

Proof: It is clear that the buyer profit under any dominant strategy **IC** and **IR** mechanism is upper bounded by the profit $\Pi(\mathbf{x}^*, \mathbf{0})$ of the point-wise optimal allocation $\mathbf{x}^*$. From (18), it follows that $(\mathbf{x}^*, \mathbf{t}^{**})$ is ex-post (pointwise) **IR**.

Thus, all that remains is to show that $(\mathbf{x}^*, \mathbf{t}^{**})$ is dominant strategy **IC**. Suppose supplier i bids $(\hat{c}_i, \hat{q}_i)$. Then, for all possible misreports $\hat{\mathbf{b}}$ of suppliers other than i, we have

$$\begin{aligned}
& t_i^{**}((\hat{c}_i, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) - \hat{c}_i x_i^*((\hat{c}_i, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) \\
&= \int_{c_i}^{\bar{c}} x_i^*((u, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) du \\
&\quad + \int_{\hat{c}_i}^{c_i} x_i^*((u, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) du - (c_i - \hat{c}_i) x_i^*((\hat{c}_i, \hat{q}_i), \widehat{\mathbf{b}}_{-i}), \\
&\leq \int_{c_i}^{\bar{c}} x_i^*((u, \hat{q}_i), \widehat{\mathbf{b}}_{-i}) du, , \qquad (19) \\
&\leq \int_{c_i}^{\bar{c}} x_i^*((u, q_i), \widehat{\mathbf{b}}_{-i}) du, , \qquad (20) \\
&= t_i^{**}(\mathbf{b}_i, \widehat{\mathbf{b}}_{-i}) - c_i x_i^*(\mathbf{b}_i, \widehat{\mathbf{b}}_{-i}),
\end{aligned}$$

where inequality (19) follows from the fact that $x_i^*((c_i, q_i), \mathbf{b}_{-i})$ is non-increasing in c_i for all $(q_i, \mathbf{b}_{-i})$ (see Lemma 2 (a)) and inequality (20) is a consequence of the fact that $x_i^*((c_i, q_i), \mathbf{b}_{-i})$ is non-decreasing in q_i for all $(c_i, \mathbf{b}_{-i})$ (see Lemma 2 (b)). Thus, truth-telling forms a dominant strategy equilibrium. ■

3.2 Optimal Mechanism in the General Case

In this section, we consider the case when Assumption 2 does not hold, i.e. the distribution of the cost and capacity does not satisfy regularity.

The optimal allocation rule is given by the solution to following optimal control problem

$$\begin{aligned}
\max_{\mathbf{x}(\mathbf{b}), \bar{\boldsymbol{\rho}}(\mathbf{q})} \quad & \mathbb{E}_{\mathbf{b}} \left[R\Big(\sum_{i=1}^n x_i(\mathbf{b}) \Big) - \sum_{i=1}^n H_i(c_i, q_i) x_i(\mathbf{b}) + \bar{\rho}_i(q_i) \right] \\
s.t \quad & 0 \leq x_i(c_i, q_i) \leq q_i && \forall i, q_i, c_i \\
& \hat{c}_i \geq c_i \Rightarrow X_i(\hat{c}_i, q_i) \leq X_i(c_i, q_i) && \forall q_i, c_i, \hat{c}_i, i \qquad (21) \\
& \hat{q}_i \geq q_i \Rightarrow \int_{c_i}^{\bar{c}} (X_i(z, q_i) - X_i(z, \hat{q}_i)) dz \leq \bar{\rho}_i(\hat{q}_i) - \bar{\rho}_i(q_i) && \forall c_i, q_i, \hat{q}_i, i \\
& 0 \leq \bar{\rho}_i(q_i) && \forall q_i, i
\end{aligned}$$

This problem is a very large scale stochastic program and is, typically, very hard to solve numerically. We characterize the solution, under a condition weaker than regularity, which we call *semi-regularity.*

We adapt the standard one dimensional ironing procedure (see, e.g. Myerson, 1981) to our problem which has a two-dimensional type space. Let $L(c_i, q_i)$ denote the cumulative density along the cost dimension, i.e.

$$L_i(c_i, q_i) = \int_{\underline{c}}^{c_i} f_i(u, q_i) du$$

Since the density $f_i(c_i, q_i)$ is assumed to be strictly positive, $L_i(c_i, q_i)$ is increasing in c_i, and hence, invertible in the c_i coordinate. Let

$$K_i(p_i, q_i) = \int_{\underline{c}}^{c_i} H_i(u, q_i) f_i(u, q_i) dt$$

where $c_i = L_i(\cdot, q_i)^{-1}(p_i)$. Let $\hat{K}_i$ denote the convex envelop of K_i along p_i, i.e.

$$\hat{K}_i(p_i, q_i) = \inf \left\{ \lambda K_i(a, q_i) + (1-\lambda) K_i(b, q_i) | a, b \in [0, L_i(\bar{c}, q_i)], \lambda \in [0,1], \lambda a + (1-\lambda) b = p_i \right\}.$$

Define ironed-out virtual cost function $\hat{H}_i(c_i, q_i)$ by setting it to

$$\hat{H}_i(c_i, q_i) = \frac{\partial \hat{K}_i}{\partial p}(p, q) \bigg|_{p_i = L_i(c_i, q_i), q_i}$$

wherever the partial derivative is defined and extending it to $[\underline{c}, \bar{c}]$ by right continuity.

Lemma 3. *The function K_i, the convex envelop $\hat{K}_i$ and the ironed-out virtual costs $\hat{H}(c_i, q_i)$ satisfy the following properties.*

(a) $\hat{H}_i(c_i, q_i)$ is continuous and nondecreasing in c_i for all fixed q_i.

(b) $\hat{K}_i(0, q_i) = K_i(0, q_i)$, $\hat{K}_i(L_i(\bar{c}, q_i), q_i) = K_i(L_i(\bar{c}, q_i), q_i)$,

(c) For all q_i and p_i, $\hat{K}_i(p_i, q_i) \leq K_i(p_i, q_i)$.

(d) Whenever $\hat{K}_i(p_i, q_i) < K_i(p_i, q_i)$, there is an interval (a_i, b_i) containing p_i such that $\frac{\partial}{\partial p}\hat{K}(p, q_i) = c$, a constant, for all $p \in (a_i, b_i)$. Thus, $\hat{H}_i(c_i, q_i)$ is constant with $c_i \in L_i(\cdot, q_i)^{-1}((a_i, b_i))$.

See (Rockafeller, 1970) for the proofs of these assertions. Now, we are ready to state our weaker regularity assumption.

Assumption 3 (Semi-Regularity)**.** *For all $i = 1, 2, \cdots, n$, the ironed out virtual marginal production cost, $\hat{H}_i(c_i, q_i)$ is non-increasing in q_i.*

From Lemma 3 (a) above, it follows that the semi-regularity implies the usual regularity of $\hat{H}_i$, i.e. $\hat{H}_i$ satisfies Assumption 2. Theorem 4 shows that if we use this ironed out virtual cost function in the buyer's profit function instead of the original virtual cost and then pointwise maximize to find the optimal allocation relaxing the monotonicity constraints on the optimal allocation and the side payments $\bar{\rho}_i$, then the resulting mechanism is incentive compatible with $\bar{\rho}_i = 0$ and revenue maximizing.

Theorem 4. *Suppose Assumption 3 holds. Let $\mathbf{x}^{\mathrm{I}} : \mathbf{B} \rightarrow \mathbb{R}_+^n$ denote any solution of the pointwise optimization problem*

$$\max_{\mathbf{0} \leq \mathbf{x} \leq \mathbf{q}} \left\{ R\Big(\sum_{i=1}^n x_i\Big) - \sum_{i=1}^n x_i \widehat{H}_i(c_i, q_i) \right\}.$$

Set the transfer payment function

$$t_i^{\mathrm{I}}(\mathbf{b}) = c_i x_i^{\mathrm{I}}(\mathbf{b}) + \int_{c_i}^{\bar{c}} x_i^{\mathrm{I}}((u, q_i), \mathbf{b}_{-i}) du. \tag{22}$$

Then $(\mathbf{x}^{\mathrm{I}}, \mathbf{t}^{\mathrm{I}})$ is a revenue maximizing, dominant strategy incentive compatible and individually rational procurement mechanism.

Proof: Let $\mathbf{x}$ be any **IC** allocation and let $\boldsymbol{\rho}$ denote the corresponding offered surplus. Define

$$\widehat{\Pi}(\mathbf{x}, \bar{\boldsymbol{\rho}}) \equiv \mathbb{E}_b \left[R\left(\sum_{i=1}^{n} x_i(b)\right) - \sum_{i=1}^{n} x_i(b)\widehat{H}_i(c_i, q_i) - \sum_{i=1}^{n} \bar{\rho}_i(q_i) \right],$$

i.e. $\widehat{\Pi}(\mathbf{x}, \bar{\boldsymbol{\rho}})$ denotes buyer profit when the virtual costs $H_i(c_i, q_i)$ are replaced by the ironed-out virtual costs $\widehat{H}_i(c_i, q_i)$. Then

$$\Pi(\mathbf{x}, \bar{\boldsymbol{\rho}}) - \widehat{\Pi}(\mathbf{x}, \bar{\boldsymbol{\rho}}) = \int_{\underline{q}}^{\bar{q}} \left[\int_{\underline{c}}^{\bar{c}} \left(\widehat{H}_i(c_i, q_i) - H_i(c_i, q_i) \right) X_i(c_i, q_i) f_i(c_i, q_i) dc_i \right] dq_i$$

The inner integral

$$\begin{aligned}
&\int_{\underline{c}}^{\bar{c}} \left(\widehat{H}_i(c_i, q_i) - H_i(c_i, q_i) \right) X_i(c_i, q_i) f_i(c_i, q_i) dc_i \\
&= \left(\hat{K}_i(c_i, t) - K_i(c_i, t) \right)\Big|_0^{L_i(c_i, q_i)} - \int_{\underline{c}}^{\bar{c}} \left(\hat{K}_i(c_i, q_i) - K_i(c_i, q_i) \right) f_i(c_i, q_i) d_{c_i}\left[X_i(c_i, q_i)\right], \\
&= -\int_{\underline{c}}^{\bar{c}} \left(\hat{K}_i(L(c_i, q_i), q_i) - K_i(L(c_i, q_i), q_i) \right) f_i(c_i, q_i) \partial_{c_i} X_i(c_i, q_i), && (23) \\
&\le 0, && (24)
\end{aligned}$$

where (23) follows from Lemma 3 (b), and (24) follows from Lemma 3 (c) and the fact that $\partial_{c_i} X_i(c_i, q_i) \le 0$ for any **IC** allocation rule. Thus, we have the $\widehat{\Pi}(\mathbf{x}, \boldsymbol{\rho}) \ge \Pi(\mathbf{x}, \bar{\boldsymbol{\rho}})$.

A proof technique identical to the one used to prove Theorem 3 establishes that $(\mathbf{x}^{\mathrm{I}}, \mathbf{t}^{\mathrm{I}})$ is an dominant strategy **IC** and **IR** procurement mechanism that maximizes the ironed-out buyer profit $\widehat{\Pi}$. Note that the corresponding offered surplus $\bar{\rho}^{\mathrm{I}}(q) \equiv 0$.

Suppose $\hat{K}_i(L(c_i, q_i), q_i) < K_i(L(c_i, q_i), q_i)$. Then Lemma 3 (d) implies that $H_i(c_i, q_i)$ is a constant for some neighborhood around c_i, i.e. $\partial_{c_i} X_i(c_i, q_i) = 0$ in some neighborhood of c_i. Consequently, the inequality (24) is an equality when $\mathbf{x} = \mathbf{x}^I$, i.e. $\widehat{\Pi}(\mathbf{x}^{\mathrm{I}}, \boldsymbol{\rho}^{\mathrm{I}}) = \Pi(\mathbf{x}^{\mathrm{I}}, \boldsymbol{\rho}^{\mathrm{I}})$. This establishes the result. ■

Theorem 4 characterizes the revenue maximization direct mechanism when the virtual costs $H_i(c_i, q_i)$ satisfy semi-regularity, or equivalently, when the ironed-out virtual costs $\hat{H}_i(c_i, q_i)$ satisfy regularity. When semi-regularity does not hold, the optimal direct mechanism can still be computed by numerically solving the stochastic program (21). Our numerical experiments lend support to the following conjecture.

Conjecture 5. *A revenue maximizing procurement mechanism has the following properties.*

(a) The side payments $\bar{\boldsymbol{\rho}} \equiv \mathbf{0}$.

(b) There exist completely ironed-out virtual cost functions $\tilde{H}_i$ *such that the corresponding point-wise solution* $\widetilde{\mathbf{x}} = \mathit{argmax}\left\{R(\sum_i \tilde{x}_i(\mathbf{b})) - \sum_{i=1}^{n} \tilde{H}_i(c_i, q_i)\tilde{x}_i(\mathbf{b}) : \mathbf{0} \le \tilde{\mathbf{x}} \le \mathbf{q}\right\}$ *is the revenue maximizing* **IC** *allocation rule.*

(c) The ironing procedure and the completely ironed-out virtual costs $\tilde{H}_i(c_i, q_i)$ *depend on the* revenue function R, *in addition to the joint prior.*

Rochet and Chone (1998) presents a general approach for multidimensional screening but in a model where the agents have both sided incentives.

3.3 Low-bid Implementation of the Optimal Auction

In this section, we assume that all the suppliers are identical, i.e. $F_i(c,q) = F(c,q)$, and that the distribution $F(c,q)$ satisfies Assumption 1 and Assumption 2. From (16) it follows that the expected transfer payment

$$T_i^*(c_i, q_i) = c_i X_i^*(c_i, q_i) + \int_{c_i}^{\bar{c}} X_i^*(u, q_i) du$$

Note that $T_i^*(c_i, q_i) = 0$, whenever $X_i^*(c_i, q_i) = 0$. Define a new point-wise transfer payment $\widetilde{\mathbf{t}}$ as follows.

$$\tilde{t}_i(\mathbf{b}) = \left(c_i + \frac{\int_{c_i}^{\bar{c}} X_i^*(t, q_i) dt}{X_i^*(c_i, q_i)} \right) x_i^*(c_i, q_i) \tag{25}$$

Then $\mathbb{E}_{(c_{-i}, q_{-i})} [t_i(c, q)] = T_i^*(c_i, q_i)$, therefore, $(\mathbf{x}^*, \tilde{\mathbf{t}})$ is Bayesian **IC** and **IR**. We use the transfer function $\widetilde{\mathbf{t}}$ to compute the bidding strategies in a "low bid" implementation of the direct mechanism. The "get-your-bid" auction proceeds as follows:

1. Supplier i bids the capacity $\hat{q}_i \leq q_i$, she is willing to provide and the marginal payment p_i she is willing to accept.
2. The buyer's actions are as follows:
   (a) Solve for the true marginal cost c_i by setting[1]
   $$p_i = \phi(c_i, \hat{q}_i) = c_i + \frac{\int_{c_i}^{\bar{c}} X_i^*(t, \hat{q}_i) dt}{X_i^*(z, \hat{q}_i)}.$$
   (b) Aggregates these bids and forms the virtual procurement cost function $\tilde{c}$ by setting
   $$\tilde{c}(q) = \sum_{j=1}^{i-1} \hat{q}_{[j]} h_{[j]} + \Big(q - \sum_{j=1}^{i-1} \hat{q}_{[j]} \Big) h_{[i]} \tag{26}$$
   for $\sum_{j=1}^{i-1} \hat{q}_{[j]} \leq q \leq \sum_{j=1}^{i} \hat{q}_{[j]}$, where as before $h_{[i]}$ denotes the i-th lowest virtual marginal cost and $\hat{q}_{[i]}$ is the capacity bid of the corresponding supplier.
   (c) Solve for the quantity $\tilde{q} = \text{argmax}\{R(q) - \tilde{c}(q)\}$. Set the allocation
   $$\tilde{x}_{[i]} = \begin{cases} \hat{q}_{[i]}, & \sum_{j=1}^{i} \hat{q}_{[j]} \leq \tilde{q}, \\ \tilde{q} - \sum_{j=1}^{i-1} \hat{q}_{[j]}, & \sum_{j=1}^{i-1} \hat{q}_{[j]} \leq q \leq \sum_{j=1}^{i} \hat{q}_{[j]}, \\ 0 & \text{otherwise.} \end{cases}$$

[1] We assume that $\phi(c_i, q_i)$ is strictly increasing in c_i, for all q_i. This would be true, for example when the virtual costs H_i are strictly increasing in c_i. Note that previously, we had been working with allocations that were non-decreasing.

3. Supplier i produces $\tilde{x}_i$ and receives $\tilde{p}_i \tilde{x}_i$.

When all the suppliers are identical, the expected allocation function $X_i^*(c, q)$ is independent of the supplier index i. We will, therefore, drop the index.

Theorem 6. *The bidding strategy*

$$\tilde{q}(c, q) = q,$$
$$\tilde{p}(c, q) = \phi(c, q) \equiv c + \frac{\int_c^{\bar{c}} X^*(t, q) dt}{X^*(c, q)},$$

is a symmetric Bayesian Nash equilibrium for the "get-your-bid" procurement mechanism.

Proof: Comparing (14) and (26), it is clear that, in equilibrium, $\mathbf{x}^*(\mathbf{b}) = \widetilde{\mathbf{x}}(\mathbf{p}, \mathbf{q})$.

Assume that all suppliers except supplier i use the bidding the proposed bidding strategy. Then the expected profit $\pi_i(\hat{p}_i, \hat{q}_i)$ of supplier i is given by

$$\begin{aligned} \pi_i(\hat{p}_i, \hat{q}_i) &= (\hat{p}_i - c_i)\tilde{X}_i(\hat{p}_i, \hat{q}_i) \\ &= (\hat{p}_i - c_i)X_i^*(\hat{c}_i, \hat{q}_i), \end{aligned}$$

where $\hat{c}_i$ given by the solution of the equation $\hat{p}_i = \phi(c, \hat{q}_i)$. Thus, we have that

$$\begin{aligned} \pi_i(\hat{p}_i, \hat{q}_i) &\leq (\hat{p}_i - c_i)X_i^*(\hat{c}_i, q_i), &(27) \\ &= \int_{\hat{c}_i}^{\bar{c}} X^*(u, q_i)du - (\hat{c}_i - c_i)X^*(c_i, q_i), \\ &= \int_{c_i}^{\bar{c}} X^*(u, q_i)du + \int_{\hat{c}_i}^{\bar{c}} X^*(u, q_i)du(\hat{c}_i - c_i)X^*(c_i, q_i), \\ &\leq \int_{c_i}^{\bar{c}} X^*(u, q_i)du = \pi(\tilde{p}(c_i, q_i), \tilde{q}(q_i)), &(28) \end{aligned}$$

where (27) and (28) follows from, respectively, Lemma 2 (b) and (a). Thus, it is optimal for supplier i to bid according to the proposed strategy. ■

3.4 Corollaries

Since the point-wise profit in (12) depends on the capacity q_i only through the conditional distribution $F_i(c_i \mid q_i)$ of the cost c_i given capacity q_i, the following result is immediate.

Corollary 1. *Suppose the marginal cost c_i and capacity q_i are independently distributed. Then the optimal allocation rule (and the corresponding transfer function) is insensitive to the capacity distribution.*

Contrasting this result with the "get-your-bid" implementation in the last section, we find that although the optimal auction mechanism is insensitive to the capacity, the supplier bidding strategies may depend on the capacity distribution.

The following result characterizes the buyers profit function when the suppliers' capacity is common knowledge.

Corollary 2. *Suppose suppliers' capacity is common knowledge. Then the buyers expected profit under any feasible,* **IC** *and* **IR** *allocation rule* $\mathbf{x}$ *is given by*

$$\Pi(\mathbf{x}) = \mathbb{E}_c \left[R\left(\sum_{i=1}^n x_i(c)\right) - \sum_{i=1}^n x_i(c)\left(c_i + \frac{F_i(c_i)}{f_i(c_i)}\right)\right] \tag{29}$$

Suppose the buyer wishes to procure a fixed quantity Q from the suppliers. Since a given realization of the capacity vector $\mathbf{q}$ can be insufficient for the needs to the buyer, i.e. $\sum_{i=1}^n q_i < Q$, we have to allow for the possibility of an exogenous procurement source. We assume that the buyer is able to procure an unlimited quantity at a marginal cost $c_0 > \bar{c}$. Let $EC(Q)$ denote the expected cost of procuring quantity Q by any optimal mechanism.

Corollary 3 (**Fixed Quantity Auction**)**.** *Suppose Assumption 1 and 2 hold. Then*

$$EC(Q) = \mathbb{E}_{(c,q)} \left\{ \begin{array}{ll} \min & \sum_{i=1}^n x_i(c,q) q_i H_i(c_i, q_i) + q_0 c_0 \\ s.t. & \sum_{i=1}^n x_i q_i + q_0 = Q \\ & 0 \le x \le q \end{array} \right\}. \tag{30}$$

Results in this paper can be adapted to other principle-agent mechanism design settings. Consider monopoly pricing with capacitated consumers. Suppose a monopolist seller with a strictly convex production cost $c(x)$ faces a continuum of customers with utility of the form

$$u(x,t;\theta,q) = \left\{ \begin{array}{ll} \theta x - t, & x \le q, \\ -\infty, & x > q, \end{array} \right.$$

where (θ, q) is the private information of the consumers. The form of the utility function $u(x,t;\theta,q)$ prevents the customer from overbidding capacity. This is necessary for the seller to be able to check individual rationality. As always the type distribution $F : [\underline{\theta}, \bar{\theta}] \times [\underline{q}, \bar{q}] \to \mathbb{R}_{++}$ is common knowledge.

Corollary 4. *Suppose the distribution* $F(\theta, q)$ *satisfies the regularity assumption that* $\nu(\theta,q) = \theta - \frac{1-F(\theta|q)}{f(\theta|q)}$ *is separately non-decreasing in both* θ *and* q*. Then the following holds for monopoly pricing with capacitated buyers.*

(a) The seller profit $\Pi(x)$ *for any feasible,* **IC** *allocation rule* x*, the seller expected profit is of the form*

$$\Pi(x) = \mathbb{E}_{(\theta,q)} \left[\Big(\theta - \frac{1-F(\theta|q)}{f(\theta|q)}\Big) x(\theta,q) - c(x(\theta,q)) \right]$$

(b) An optimal direct mechanism is given by the allocation rule

$$x^*(\theta,q) = \underset{0 \le x \le q}{argmax} \left[\Big(\theta - \frac{1-F(\theta|q)}{f(\theta|q)}\Big) x - c(x) \right]$$

and transfer payment

$$t^*(\theta,q) = \int_{\underline{\theta}}^{\theta} x^*(t,q) dt$$

Since the type space is two-dimensional, the optimal direct mechanism can be implement by a posted tariff only if the parameter θ and the capacity q are independently distributed.

All our results in this section easily extend to nonlinear convex production cost $c_i(\theta, x)$, $\theta \in [\underline{\theta}, \bar{\theta}]$, that are super-linear, i.e. $\frac{\partial^2 c_i}{\partial\theta\partial x} > 0$. In this case, the virtual production cost $H_i(\theta_i, x)$ is given by

$$H_i(\theta_i, x) = c_i(\theta_i, q_i, x) + c_{i\theta}(\theta_i, x)\frac{F_i(\theta_i|q_i)}{f_i(\theta_i|q_i)}.$$

4 Conclusion and Extensions

This paper proposes a procurement mechanism that is able to optimally screen for both privately known capacities and privately known cost information. The results can be easily adapted to other principle-agent mechanism design problems in which agents have a privately known bounds on consumption. In Iyengar and Kumar (2006) we show that our model extends to reverse auction with multiple products when the private information about the cost is one dimensional. In this report, we also present an application of the multi-product model to auctioning multi-period supply contract in which a buyer who faces the risk of variable capacities over time can effectively hedge this risk by committing to order from different suppliers in different periods.

Two very simple natural extensions of the model proposed here lead to hard mechanism design problems:

(a) Suppose the suppliers can purchase additional capacity at a cost. Then supplier's utility explicitly depends on the initial capacity and mechanism design problem is truly 2-dimensional. Thus, Lemma 1 fails to holds and the mechanism design problem does not appear to have any tractable formulation.

(b) Suppose the private information about the product cost in the multi-product case is multi-dimensional. In this case even the uncapacitated version of this problem remains unsolved.

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